



Soret and Dufour Effects on MHD Casson Fluid Over a Vertical Plate in Presence of Chemical Reaction and Radiation

N. Ananda Reddy¹, K. Janardhan¹

Department of Mathematics, Annamacharya Institute of Technology Sciences, Rajampet, Kadapa (Dist), A.P, India.

ABSTRACT

The combined effects of chemical reaction, radiation, Dufour and Soret effects on Casson MHD fluid flow over a vertical plate with heat source/sink has been studied with constant velocity, variable temperature and concentration. The governing equations of the flow have been solved by using the finite difference method. The effects of various parameters on velocity, temperature and concentration fields are studied with the help of graphs. It is found that velocity decreased with the increase of Casson parameter, Hartmann number, thermal conductivity, Schmidt number, Chemical parameter and decrease of Soret effect, Thermal Grashof number and Mass Grashof number. Concentration found to be decreasing with the decrease of Soret number and increasing chemical parameter. Temperature decreased when thermal radiation is increased and heat source is decreased.

Key Words: Casson fluid, Dufour effect, Soret effect, MHD, Chemical reaction

INTRODUCTION

Casson fluid exhibits yield stress. It is well known that Casson fluid is a shear thinning liquid which is assumed to have an infinite viscosity at zero rate of shear, a yield stress below which no flow occurs, and a zero viscosity at an infinite rate of shear, i.e., if a shear stress less than the yield stress is applied to the fluid, it behaves like a solid, whereas if a shear stress greater than yield stress is applied, it starts to move. Casson fluids like Honey, human blood, jelly, sauces, etc., are having significant importance. Human blood can also be treated as Casson fluid.

In recent days, Casson fluids have occupied a prominence role in various fields like biological, chemical, medical and engineering applications. In industrial environment, Various researchers have carried out their research work in this direction enlightening the effects of various parameters on the flow of the Casson fluid. Mustafa and Khan studied the MHD flow and heat transfer of Casson nanofluid over a non-linearly stretching sheet with non-linear temperature distribu-

tion. The heat and mass transfer characteristics of a magneto hydrodynamic Casson fluid flow on a parallel plate channel having stretching walls, when subjected to a uniform transverse magnetic field are analyzed by Sarojamma et al. Swati et al numerically examined the boundary layer flow due to an exponentially stretching surface in the presence of an applied magnetic field. Casson fluid model is used to characterize the non-Newtonian fluid behavior. The flow is subjected to suction/blowing at the surface. Analysis is carried out in presence of thermal radiation and prescribed surface heat flux. Prakash et al studied the MHD two-dimensional boundary layer flow of a Casson fluid in the presence of chemical reaction and thermal radiation accompanied by heat and mass transfer towards an exponentially stretching sheet.

Animasaun et al. studied the thermal conductivity and motion of temperature dependent plastic dynamic viscosity of steady incompressible laminar free convective MHD Casson fluid flow over an exponentially stretching surface with suction and exponentially decaying internal heat generation.

Corresponding Author:

N. Ananda Reddy, Department of Mathematics, Annamacharya Institute of Technology Sciences, Rajampet, Kadapa (Dist), A.P, India.
E-mail: anandareddy@gmail.com

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The effects of thermal radiation, suction/blowing, viscous dissipation, heat source/sink and chemical reaction on boundary layer flow of a non-Newtonian Casson fluid in presence of heat and mass transfer towards a porous exponentially stretching sheet with velocity slip and thermal slip conditions were analyzed by Saidulu and Lakshmi. The steady flow and heat transfer of Casson fluid from a permeable horizontal cylinder in the presence of slip condition in a non-Darcy porous medium is analyzed by maintaining the cylinder surface at a constant temperature is studied by Prasad et al. Shehzad et al derived a series solution for the effect of mass transfer in the MHD flow of a Casson fluid over a porous stretching sheet in the presence of a chemical reaction. Casson fluid flow over a vertical porous surface with chemical reaction in the presence of hydromagnetic field has been studied by Arthur et al. The heat and mass transfer effect in a boundary layer flow of an electrically conducting viscous fluid subject to transverse magnetic field past a moving vertical plate through porous medium in the presence of heat source and chemical reaction is analysed by Tripathy et al.

The aim of the present paper is to analyze the combined effects of chemical, radiation, Dufour and Soret parameters on Casson MHD fluid flow over a vertical plate with heat source/ sink has been studied with constant velocity, variable temperature and concentration. The governing equations of the flow have been solved by using the finite difference method. The effects of various parameters on velocity, temperature and concentration fields are studied with the help of graphs.

Formulation of the problem

Consider a two-dimensional steady Casson fluid of incompressible, viscous, electrically conducting fluid over a vertical plate moving with constant velocity with radiation and chemical reaction in the presence of Soret and Dufour is considered. Consider the flow of an incompressible viscous fluid passing a flat sheet coinciding with plane $y = 0$. We select the Cartesian coordinate system such that the x - axis be taken parallel to the surface and y is perpendicular to the surface. The fluid occupies a half space $y > 0$. The flow is subjected to a constant applied magnetic field B_0 in the y -direction. The magnetic Reynolds number is considered to be very small so that the induced magnetic field is negligible in comparison to the applied magnetic field. The surface temperature of the plate oscillates with small amplitude about a nonuniform mean temperature. The fluid is assumed to have constant properties except for the influence of the density variations with temperature and concentration which are considered only in the body force term. The temperature of the plate oscillates with little amplitude about a non-uniform temperature.

The rheological equation of state for an isotropic flow of a Casson fluid [000] can be expressed as:

$$\tau_{ij} = \begin{cases} \left(\frac{\mu_b + p_y}{\sqrt{2\pi}} \right) 2e_{ij}, & \pi > \pi_c \\ \left(\frac{\mu_b + p_y}{\sqrt{2\pi}} \right) 2e_{ij}, & \pi < \pi_c \end{cases} \quad (1)$$

In Equation (1), $\pi = e_{ij} e_{ij}$, where e_{ij} is the $(i, j)^{th}$ component of deformation rate. This means that π is the product of the component of deformation rate with itself. Also, π_c is a critical value of this product based on the non-Newtonian model, μ_b is the plastic dynamic viscosity of the non-Newtonian fluid and P_y is the yield stress of the fluid. The equations governed the unstudied boundary layer flow of the Casson fluid is

$$\frac{\partial u^*}{\partial t^*} = \left(1 + \frac{1}{\beta} \right) \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta(T^* - T_\infty) + g\beta(C^* - C_\infty) - \frac{\sigma}{\rho} B_0^2 u^* \quad (2)$$

$$\frac{\partial T^*}{\partial t^*} = \frac{k}{\rho C_p} \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho C_p} \frac{\partial q_r^*}{\partial y^*} + \frac{Q_0}{\rho C_p} (T^* - T_\infty) + \frac{D_m K_t}{C_p C_\infty} \frac{\partial^2 C^*}{\partial y^{*2}} \quad (3)$$

$$\frac{\partial C^*}{\partial t^*} = D \frac{\partial^2 C^*}{\partial y^{*2}} - K_r (C^* - C_\infty) + \frac{D_m K_t}{T_m} \frac{\partial^2 T^*}{\partial y^{*2}} \quad (4)$$

Equations (2), (3) and (4) refers Momentum Equation, Energy Equation and Species Equation respectively where u is the velocity of the fluid, β is Casson parameter, Q_0 is the heat source/sink parameter, D is the molecular diffusivity, k is thermal conductivity, C is mass concentration, t is time, ν is the kinematics viscosity, g is the gravitational constant, β and β^* are the thermal expansions of fluid and concentration, ρ is density, c_p is the specific heat capacity at constant pressure, D_m Coefficient of mass diffusivity, k_t Thermal diffusion ratio, T_m Mean fluid temperature, T_∞ Free stream temperature of the surrounding fluid, C_∞ Free stream concentration, T Fluid temperature, C Fluid concentration y is distance, q_r is the radiative flux, β is the magnetic field, k_r is the chemical reaction rate constant.

R.H.S. of equation (2), second term is thermal heat effect, third term is thermal concentration effect, fourth term is magnetic effect, and second term is thermal buoyancy effect. R.H.S. of equation (3) second term is thermal radiation flux and third term is thermal radiation and fourth term is Dufour effect. R.H.S. of equation (4), second term is chemical reaction and third term Soret effect. Under the above assumptions the physical variables are functions of y and t . The boundary conditions for the velocity, temperature and concentration fields are:

$$\begin{aligned} u^* &= 0, & T^* &= T_\infty, & C^* &= C_\infty & \text{for all } y^*, t^* \leq 0 \\ u^* &= U, & T^* &= T^* + \varepsilon(T^* - T_\infty)e^{i\omega t^*}, & C^* &= C^* + \varepsilon(C^* - C_\infty)e^{i\omega t^*} & \text{at } y=0 \\ u^* &\rightarrow 0, & T^* &\rightarrow 0, & C^* &\rightarrow 0 & \text{as } y \rightarrow \infty \end{aligned} \quad (5)$$

Introducing the dimensionless quantities with thermal radiation flux gradient expressed and we assume that the temperature differences within the flow are sufficiently small so that

T^{*4} can be expressed as a linear function of T^* after using Taylor's series to expand T^{*4} about the free stream temperature T_∞^* and neglecting higher-order terms. This results in the following approximation.

$$\frac{\partial q_r^*}{\partial y^*} = -4a^* \sigma^* (T_\infty^{*4} - T^{*4}) \text{ and } T^{*4} \cong 4 T_\infty^{*3} T^* - 3T_\infty^{*4} \quad (6)$$

The following dimensionless quantities are introduced

$$u = \frac{u^*}{U}, y = \frac{U y^*}{\nu}, t = \frac{U t^*}{\nu}, \theta = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*}, \phi = \frac{C^* - C_\infty^*}{C_w^* - C_\infty^*}, Q = \frac{Q_0 \nu}{\rho C_p U^2}, K = \frac{K^* u_0^2}{\nu^2},$$

$$P_r = \frac{\mu C_p}{k}, S_c = \frac{\nu}{D}, M = \frac{\sigma B_0^2 \nu}{\rho U^2}, G_r = \frac{\nu \beta g (T_w^* - T_\infty^*)}{U^3}, G_c = \frac{\nu \beta^* g (C_w^* - C_\infty^*)}{U^3},$$

$$K_r = \frac{K_r^* \nu}{U_0^2}, R = \frac{16 a^* \nu \sigma^* T_\infty^{*3}}{\rho C_p U^2}, \mu = \nu \rho, D_0 = \frac{D_m K_t}{C_p C_i \nu (T_w^* - T_\infty^*)}, S_r = \frac{D_e K_t U^2}{T_m \nu^2} (T_w^* - T_\infty^*) \quad (7)$$

The thermal radiation flux gradient may be expressed as follows

$$-\frac{\partial q_r}{\partial y^*} = 4a \sigma^* (T_\infty^{*4} - T^{*4}) \quad (8)$$

Considering the temperature difference by assumption within the flow are sufficiently small such that T^{*4} may be expressed as a linear function of the temperature. This is attained by expanding in T^{*4} Taylor's series about T^{*4} and ignoring higher orders terms.

$$T^{*4} \cong 4 T_\infty^{*3} T^* - 3T_\infty^{*4} \quad (9)$$

Substituting the dimensionless variables (7) into (2) to (4) and using equations (8) and (9), reduce to the following dimensionless form.

$$\frac{\partial u}{\partial t} = \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 u}{\partial y^2} + G_r \theta + G_c \phi - Mu \quad (10)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial y^2} - R\theta + Q\theta + D_0 \frac{\partial^2 \phi}{\partial y^2} \quad (11)$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{S_c} \frac{\partial^2 \phi}{\partial y^2} - K_r \phi + S_r \frac{\partial^2 \theta}{\partial y^2} \quad (12)$$

The corresponding boundary conditions are

$$u = 1, \theta = 1 + \varepsilon e^{nt}, \phi = 1 + \varepsilon e^{nt} \text{ at } y = 0$$

$$u = 1, \theta = 1 + \varepsilon e^{nt}, \phi = 1 + \varepsilon e^{nt} \text{ at } y = 0$$

$$u \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0 \text{ as } y \rightarrow \infty \quad (13)$$

Where G_r is thermal Grashof number, P_r is the prandtl number, k_r is the chemical reaction parameter, R is the thermal radiation conduction number, M is Hartmann number, G_c is the mass Grashof number, Q is the heat source/sink parameter and S_r is the Soret number.

Method of Solution

Equations (10)-(12) are linear partial differential equations and are to be solved with the initial and boundary conditions (13). In fact the exact solution is not possible for this set of equations and hence we solve these equations by finite-difference method. The equivalent finite difference schemes of equations for (10)-(12) are as follows:

$$\frac{u_{i,j+1} - u_{i,j}}{\Delta t} = \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{(\Delta t)^2} + G_r \theta_{i,j} + G_c \phi_{i,j} - Mu_{i,j} - \frac{1}{K} u_{i,j} \quad (14)$$

$$\frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta t} = \frac{\theta_{i-1,j} - 2\theta_{i,j} + \theta_{i+1,j}}{(\Delta t)^2} + R\theta_{i,j} + Q\theta_{i,j} - D_0 Mu_{i,j} - \frac{1}{K} \frac{\phi_{i-1,j} - 2\phi_{i,j} + \phi_{i+1,j}}{(\Delta t)^2} \quad (15)$$

$$\frac{\phi_{i,j+1} - \phi_{i,j}}{\Delta t} = \frac{\phi_{i-1,j} - 2\phi_{i,j} + \phi_{i+1,j}}{(\Delta t)^2} + K_r \phi_{i,j} - S_r \frac{\theta_{i-1,j} - 2\theta_{i,j} + \theta_{i+1,j}}{(\Delta t)^2} \quad (16)$$

Here, the suffix 'i' refer to y and 'j' to time. The mesh system is divided by taking $\Delta y = 0.1$. From the initial condition in (13), we have the following equivalent:

$$u(i, 0) = 0, \theta(i, 0) = 0, C(i, 0) = 0 \text{ for all } i \quad (17)$$

The boundary conditions from (13) are expressed in finite-difference form as follows

$$u(0, j) = 1, \theta(0, j) = 1 + \varepsilon e^{n(j-1)\Delta t}, \phi(0, j) = 1 + \varepsilon e^{n(j-1)\Delta t} \text{ for all } j$$

$$(0, j) = 0, \theta(0, j) = 0, \phi(0, j) = 0 \text{ for all } j \quad (18)$$

(Here i_{\max} was taken as 200)

The velocity at the end of time step viz, $u(i, j+1)$ ($i=1, 200$) is computed from (14) in terms of velocity, temperature and concentration at points on the earlier time-step. After that $\theta(i, j+1)$ is computed from (15) and then $C(i, j+1)$ is computed from (16). The procedure is repeated until $t = 0.5$ (i.e. $j = 500$). During computation Δt was chosen as 0.001.

Skin-friction:

The skin-friction in non-dimensional form is given by the relation

$$\tau = -\left(\frac{du}{dy}\right)_{y=0}, \text{ where } \tau = \frac{\tau^l}{\rho U_0^2}$$

Rate of heat transfer:

The dimensionless rate of heat transfer in terms of Nusselt number is given by

$$Nu = -\left(\frac{d\theta}{dy}\right)_{y=0}$$

Rate of mass transfer:

The dimensionless rate of mass transfer in terms of Sherwood number is given by

$$Sh = -\left(\frac{dC}{dy}\right)_{y=0}$$

RESULTS AND DISCUSSION

The effects of various parameters such as Casson Parameter " β ", Thermal Grashof Number " G_r ", Mass Grashof Number " G_r ", Thermal Conductivity " K ", chemical reaction parameter k_r , Prandtl Number " P_r " Schmidt Number " S_c " Thermal radiation conduction number " R ", Hartmann number M , Heat source/sink parameter Q and Soret number S_r on the velocity, temperature and concentration fields are studied numerically and represented the results through graphs. The influence of Casson parameter, Hartmann number, thermal conductivity, Schmidt number and chemical reaction parameter on velocity is shown in the figures 1,5,6,8,9. From these figures, it is recognized that the velocity decreases with the increasing of these parameters respectively. The influence of Soret number, thermal Grashof number, the mass Grashof number on velocity are shown in the figures 2,3,4. It is clear that the velocity decreases with the decrease of these parameters respectively. The effect of Prandtl number on the velocity is shown in the figure 7. It is observed that the velocity decreases near the plate and increases far away the plate with the falling of the Prandtl number. The effect of Soret number on the concentration field is illustrated in figure 10. As the Soret number decreases the concentration is found to be decreasing. The effect of chemical reaction parameter on the concentration field is illustrated in figure 12. It is clear that the concentration is decreasing for increasing chemical reaction parameter. The effect of thermal radiation conduction number on concentration field is shown in figure 11. It is noted that the concentration decreases near the plate and increases far away the plate with falling thermal radiation conduction number. Figure 13 shows the variations of thermal radiation conduction number on temperature. It is found that the temperature is decreased when R is increased. Figure 14 shows the variation of the heat source sink parameter on temperature. It is clear that the temperature decreases when Q decreases. In Figure 15, the skin friction decreases when thermal Grashof number increases and figure 16 shows that the skin friction decreases when Soret number increases.

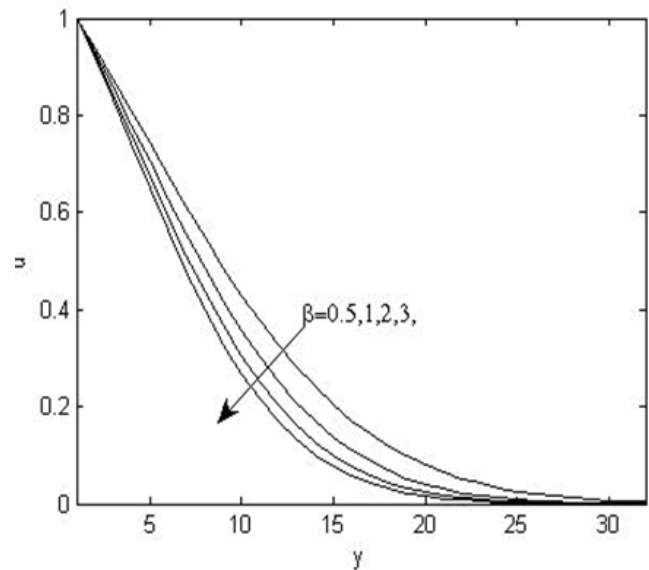


Figure 1: Effects of Casson Parameter β on velocity.

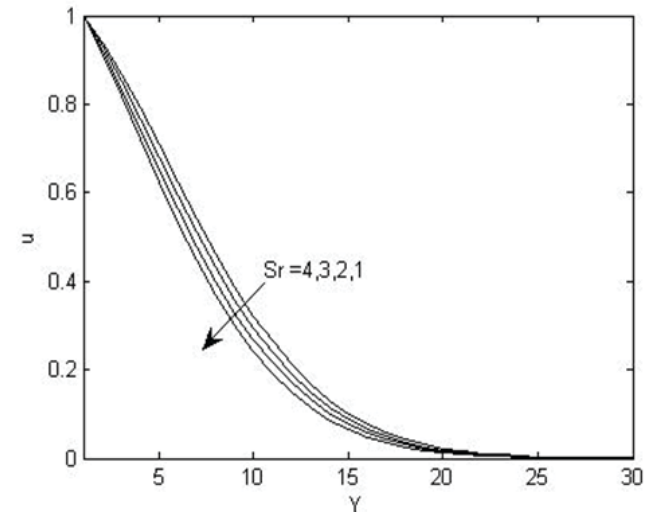


Figure 2: Effects of Soret Number S_r on velocity.

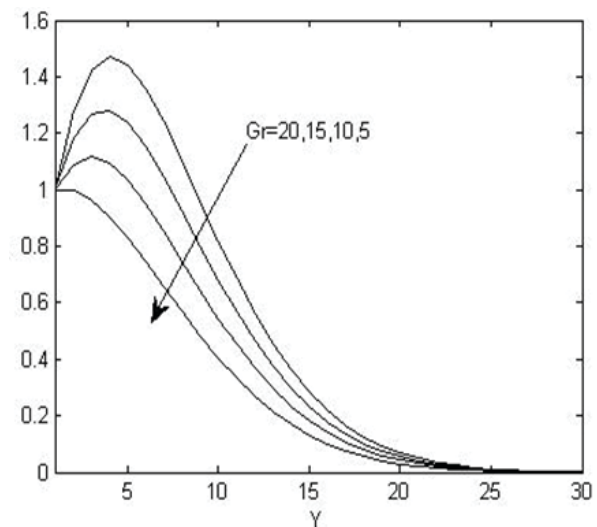


Figure 3: Effect of Thermal Grashof Number G_r on velocity.

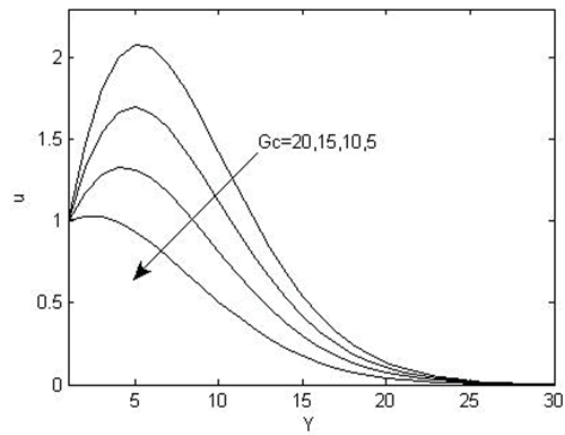


Figure 4: Effect of Mass Grashof Number G_c on velocity.

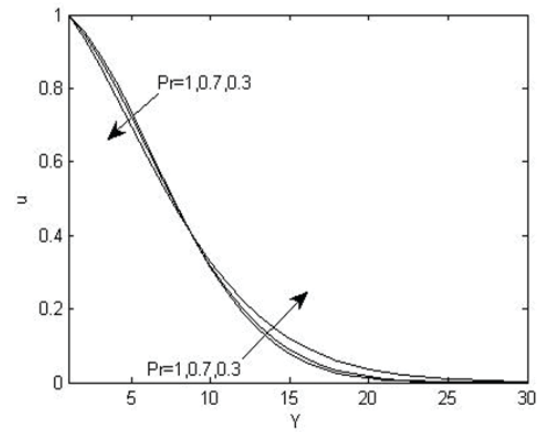


Figure 7: Effect of Prandtl Number Pr on Velocity.

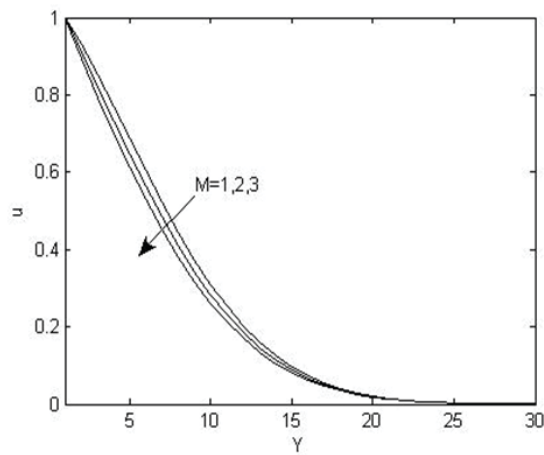


Figure 5: Effect of Hartman Number M on velocity.

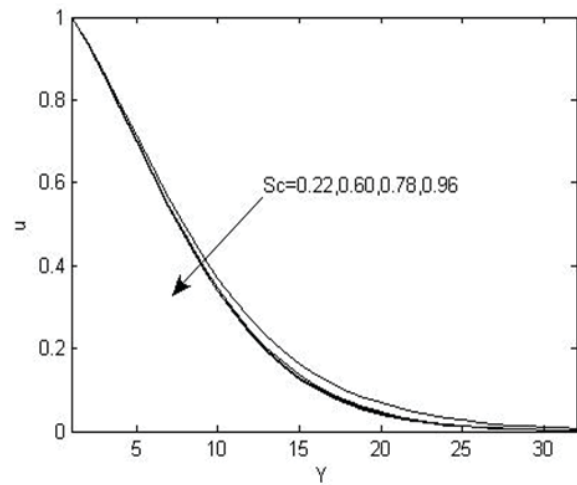


Figure 8: Effect of Schmidt Number Sc on Velocity.

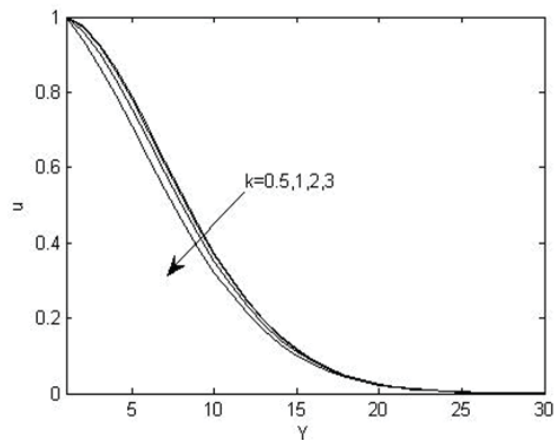


Figure 6: Effect of Thermal Conductivity K on Velocity.

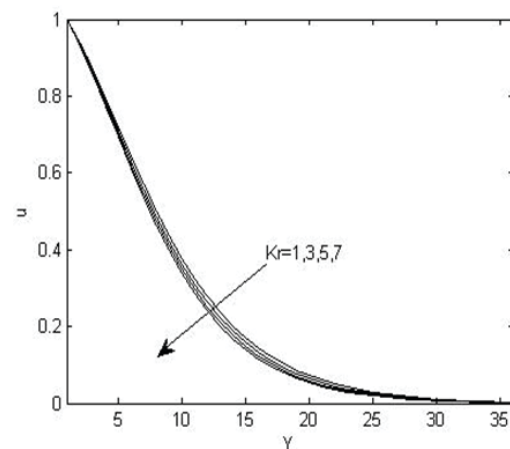


Figure 9: Effect of Chemical Reaction parameter K_r on Velocity.

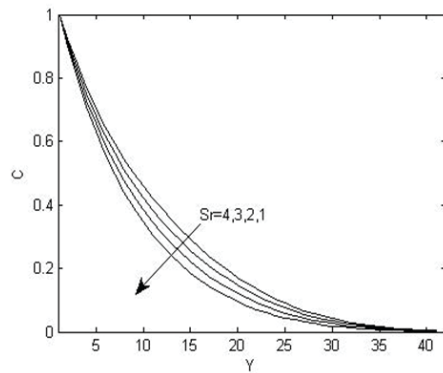


Figure 10: Effect of Soret Number P_r on Velocity.

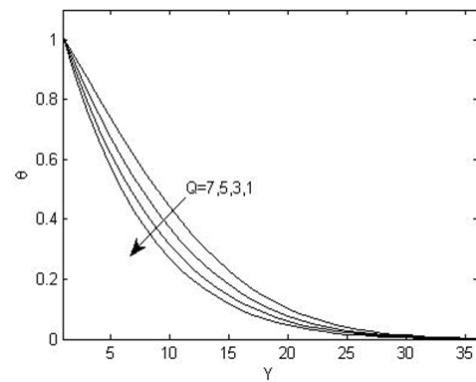


Figure 14: Effect of Heat Source /Sink Parameter Q on Temperature.

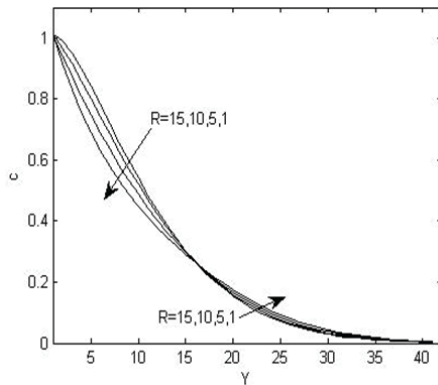


Figure 11: Effect of Prandtl Number P_r on Concentration.

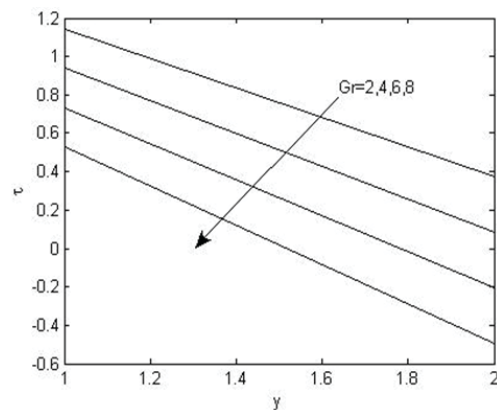


Figure 15: Effect of Thermal Grashof Number G_r on Skin Friction.

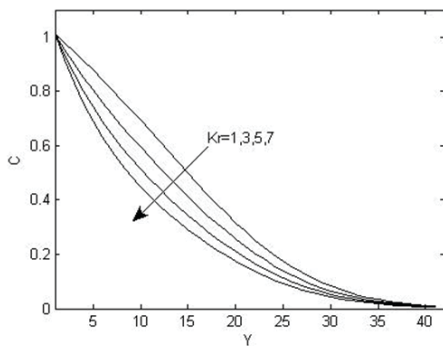


Figure 12: Effect of Chemical Reaction parameter K_r on Concentration.

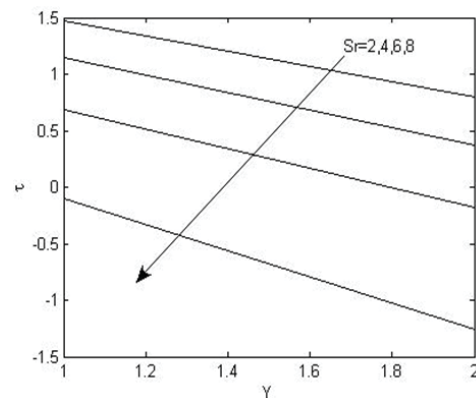


Figure 16: Effect of Soret Number S_r on Skin Friction.

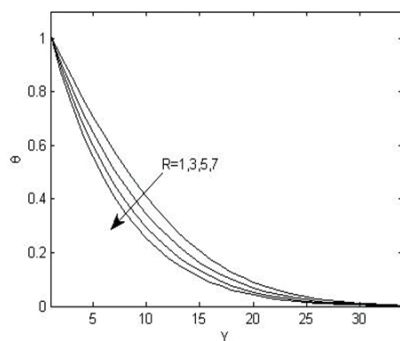


Figure 13: Effect of Thermal Radiation Conduction Number R on Temperature.

CONCLUSION

In this paper, from the study of the combined effects of chemical reaction, radiation, Dufour and Soret effects on Casson MHD fluid flow over a vertical plate with heat source / sink, it is concluded that

- Velocity decreased with the increase of Casson parameter, Hartmann number, thermal conductivity, Schmidt number, Chemical parameter and decrease of Soret effect, Thermal Grashof number and Mass Grashof number.
- Concentration is found to be decreasing with the decrease of Soret number and increasing chemical parameter.
- Temperature is decreased when thermal radiation is increased and heat source is decreased
- Skin friction decreased when thermal Grashof number and Soret number are increased

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